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CERTAIN PROBLEMS OF THE THEORY OF HYPERSONIC FLOW OF A VISCOUS HEAT-CONDUCTING GAS

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INTRODUCTION

It is well known that the effects of the viscosity and thermal conductivity of a gas in the flow field about a body moving at high supersonic speed may be of great significance. For certain flight modes, these effects may become apparent in the form of an interaction of the boundary layer with the external "inviscid" flow.[1] For still higher flight velocities and flight altitudes, it may become necessary to investigate the entire flow field on the basis of the complete Navier-Stokes equations for a viscous heat-conducting gas. The problem of analyzing and solving these equations is known to involve great mathematical difficulties. Only a few exact solutions of these equations are known at present, all of which refer to the simplest cases of motion. The present work deals with the study of certain new problems associated with the motion of a viscous heat-conducting gas, on the basis of the complete Navier-Stokes equations for the limiting case where the Mach number is infinitely large.

In the investigation of real gases, the transition to an infinitely large Mach number should always be considered as a process in which the speed of sound in undisturbed flow tends to zero while the flow speed remains fixed.[1] Hence, the flow of a viscous heat-conducting gas of very high supersonic speed may be treated as a flow with zero absolute temperature in the unperturbed region.*

A basic feature of such flows, as shown in Ref. 2, is that instead of an asymptotic attenuation of the disturbances at infinity, there is always a surface (front) that separates the region of disturbed motion from the remainder of the space occupied by the undisturbed gas. An explanation for this is that the viscosity and heat conductivity of the gas are functions of the temperature, which decrease and become zero with the latter. For flows within shock waves, such a

* This result constitutes a generalization, for the case of a real gas, of the known gas-dynamic principle stating the nondependence upon Mach number of a flow of very high supersonic speed.[1]

surface is, of course, the advancing front of a shock wave propagating through the gas at rest.

The solutions discussed in the present work relate primarily to flows in high-intensity shock waves. The study of the structure of a strong shock wave allows, in particular, to define the behavior of the solution near the front, which in the general case is a singular solution. The discussion includes also certain self-similar nonsteady motions of a gas, the self-similarity of the motions being defined by their limiting state, i.e., the zero temperature in the unperturbed region. Formally, this is associated with the fact that, in this case, the number of the dimensional parameters of the problem decreases. A detailed analysis is given of only one such problem: the uniformly accelerated motion of an infinite plate. The solution of this problem with inclusion of the numerical results appears to be the first *exact solution* of the Navier-Stokes equations obtained for the case of the flow of a viscous compressible gas about a body.

In all cases the gas is considered as an ideal gas with constant specific heat, constant Prandtl number, and an exponential dependence of viscosity upon temperature.

STRUCTURE OF A STRONG SHOCK WAVE, AND BEHAVIOR OF SOLUTIONS NEAR THE FRONT SURFACE

To gain insight into the characteristic singularities of the solution behavior near the front, and to obtain certain relations of significance for the following discussion, let us first examine the simplest type of motion of a viscous heat-conducting gas, i.e., a one-dimensional steady motion. To this belongs the thoroughly investigated flow within a shock wave that propagates through a gas at rest.[3] Let us investigate this flow on the assumption that the Mach number of the unperturbed stream is infinitely large. Let the velocity of the unperturbed portion of the flow be V_∞, and the density be ρ_∞; while the pressure p_∞, the temperature T_∞, and the enthalpy h_∞ are equal to zero. The angle β between the normal to the front surface (Fig. 1) and the velocity vector $\vec{V}_\infty$ we will assume, for conformity, as not equal to zero (oblique shock wave). The axis y in the Cartesian system of coordinates we postulate along the normal to the front. We denote the components of the velocity vector of the flow under investigation through u, v, and the pressure, density, and enthalpy through p, ρ, and h, respectively.

The law that governs the dependence of on viscosity enthalpy we write in the form:

$$\mu = Ch^n \tag{1}$$

where C is a constant. From the dimensional constants ρ_∞, V_∞, C which are included in the conditions of the problem, may be derived the quantity l, having the dimensionality of length.†

† Quantity l is of the order of the length of the free path of molecules behind a shock wave.

$$l = \frac{C}{\rho_\infty} V_\infty^{\,2n-1} \tag{2}$$

We introduce the nondimensional independent variable $\bar{y}$ and the nondimensional unknown functions

$$\begin{aligned} \bar{y} &= \frac{y}{l\cos^{2n-1}\beta}; \qquad \bar{u} = \frac{u}{V_\infty \sin\beta}, \qquad \bar{v} = \frac{v}{V_\infty \cos\beta} \\ \bar{p} &= \frac{p}{\rho_\infty V_\infty^2 \cos^2\beta}, \qquad \bar{\rho} = \frac{\rho}{\rho_\infty}, \qquad \bar{h} = \frac{h}{V_\infty^2\cos^2\beta} \end{aligned} \tag{3}$$

The equations of the one-dimensional steady motion for these variables can be written in the form

$$\begin{aligned} &\overline{\rho v}\,\frac{d\bar{u}}{d\bar{y}} = \frac{d}{d\bar{y}}\left(\bar{h}^n \frac{d\bar{u}}{d\bar{y}}\right) \\ &\overline{\rho v}\,\frac{d\bar{v}}{d\bar{y}} + \frac{d\bar{p}}{d\bar{y}} = \frac{4}{3}\frac{d}{d\bar{y}}\left(\bar{h}^n \frac{d\bar{v}}{d\bar{y}}\right) \\ &\overline{\rho v}\,\frac{d\bar{h}}{d\bar{y}} = \bar{v}\frac{d\bar{p}}{d\bar{y}} + \frac{1}{\sigma}\frac{d}{d\bar{y}}\left(\bar{h}^n \frac{d\bar{h}}{d\bar{y}}\right) + \frac{4}{3}\bar{h}^n\left(\frac{d\bar{v}}{d\bar{y}}\right)^2 + \bar{h}^n\left(\frac{d\bar{u}}{d\bar{y}}\right)^2 \\ &\frac{d(\overline{\rho v})}{d\bar{y}} = 0, \qquad \bar{p} = \frac{\gamma-1}{\gamma}\bar{\rho}\bar{h} \end{aligned} \tag{4}$$

Here, σ is the Prandtl number, and $\gamma = C_p/C_v$ is the specific-heats ratio of the gas. The boundary conditions of the problem are the following:

$$\bar{u} = \bar{v} = \bar{\rho} = 1, \qquad \bar{p} = \bar{h} = 0 \quad at \quad \bar{y} \to -\infty \tag{5}$$

the solution is limited at $\vec{y} \to +\infty$.

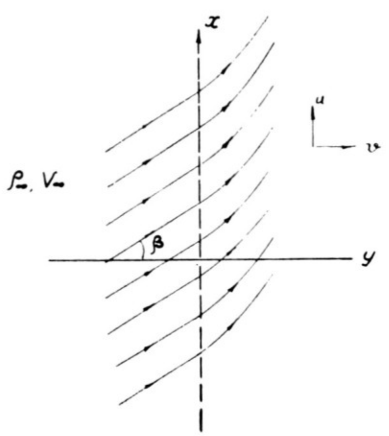

Fig. 1.

As is known,[3] the system of equations (4) can be completely integrated for $\sigma = 3/4$. If, here, $n = 1$, then its particular solution that satisfies Eq. (5) is written in the elementary form[2]:

$$\begin{aligned}
\bar{u} &= 1 \\
\bar{y} &= \frac{2}{3}(1+\epsilon)\left[(1+\epsilon)(1-\bar{v}) + \epsilon(1+\epsilon) h \frac{1-\epsilon}{\bar{v}-\epsilon} + \frac{1-\bar{v}^2}{2}\right] \\
\bar{\rho} &= \frac{1}{\bar{v}}, \qquad \bar{h} = \frac{1-\bar{v}^2}{2}, \qquad \bar{p} = \frac{\gamma-1}{\gamma}\bar{\rho}\bar{h} \\
\epsilon &= \frac{\gamma-1}{\gamma+1}
\end{aligned} \tag{6}$$

The solution for $\gamma = 1.4$ is given in Fig. 2.

Analysis of the obtained relations indicates that the perturbed portion of the flow is separated from the uniform incident flow by a front which (by selection of the unessential arbitrary constant of integration) coincides with the axis $\bar{y} = 0$. In the $n = 1$ case under consideration, the front surface is the discontinuity surface of the derivatives having finite nonzero values at $\bar{y} = +0$.

In the general case of an arbitrary Prandtl number and an arbitrary positive n, a solution in closed form cannot be obtained. However, its behavior near the front surface can be investigated by rather simple means. It is readily shown that at $\bar{y} \to 0$, the exact equation (4) can be reduced to the approximate equations of the form:

$$\begin{aligned}
\frac{d\bar{u}}{d\bar{y}} &\cong \frac{d}{d\bar{y}}\left(\bar{h}^n \frac{d\bar{u}}{d\bar{y}}\right) \\
\frac{d\bar{v}}{d\bar{y}} &\cong -\frac{d\bar{p}}{d\bar{y}} + \frac{4}{3}\frac{d}{d\bar{y}}\left(\bar{h}^n \frac{d\bar{v}}{d\bar{y}}\right) \\
\frac{d\bar{h}}{d\bar{y}} &\cong \frac{d\bar{p}}{d\bar{y}} + \frac{1}{\sigma}\frac{d}{d\bar{y}}\left(\bar{h}^n \frac{d\bar{h}}{d\bar{y}}\right) \\
\frac{d\bar{v}}{d\bar{y}} + \frac{d\bar{p}}{d\bar{y}} &\cong 0, \qquad \bar{p} = \frac{\gamma-1}{\gamma}\bar{h}
\end{aligned} \tag{7}$$

Integrated with allowance for (5), they yield the following approximations to the unknown functions at $\bar{y} \to 0$:

$$\bar{u} \cong 1 + \bar{A}\bar{y}^{\gamma/n\sigma}$$

$$\bar{v} \cong 1 - \frac{\gamma-1}{\gamma - \frac{4}{3}\sigma}\left(\frac{n\sigma}{\gamma}\bar{y}\right)^{1/n} + \bar{B}\bar{y}^{3\gamma/4n\sigma}$$

$$\bar{\rho} \cong 1 + \frac{\gamma - 1}{\gamma - \frac{4}{3}\sigma}\left(\frac{n\sigma}{\gamma}\bar{y}\right)^{1/n} - \bar{B}\bar{y}^{3\gamma/4n\sigma} \tag{8}$$

$$\bar{h} \cong \left(\frac{n\sigma}{\gamma}\bar{y}\right)^{1/n}, \qquad \bar{p} \cong \frac{\gamma - 1}{\gamma}\left(\frac{n\sigma}{\gamma}\bar{y}\right)^{1/n}$$

where $\bar{A}$ and $\bar{B}$ are arbitrary constants of integration. It must be noted that the formulas [Eq. (8)] can be used to obtain an approximation of the solution near the front in any problems involving curvilinear and unsteady shock waves, because a small element of the front surface may always be considered plane, and the velocity of its propagation during a sufficiently short period of time may be considered constant.

For the special case of a plane steady shock wave, the constants $\bar{A}$ and $\bar{B}$ are equal to zero.

From the derived equations (8), it can be seen that in the general case, the solution near the front has a singular behavior.

SELF-SIMILAR MOTIONS OF A VISCOUS HEAT-CONDUCTING GAS

If the gas temperature in the unperturbed region is zero, then the density ρ_∞ is the only thermodynamic parameter that determines the state of the gas in this region. The other dimensional parameter of the problem is the constant C in the law of the dependence of viscosity on enthalpy (1). The dimensions of these equations are as follows:

$$[\rho_\infty] = \frac{M}{L^3} \qquad [C] = \frac{MT^{2n-1}}{L^{2n+1}} \tag{9}$$

Here, L is the symbol for the length unit, T for the time unit, and M for the mass unit. If there are no parameters of the problem whose dimensions are

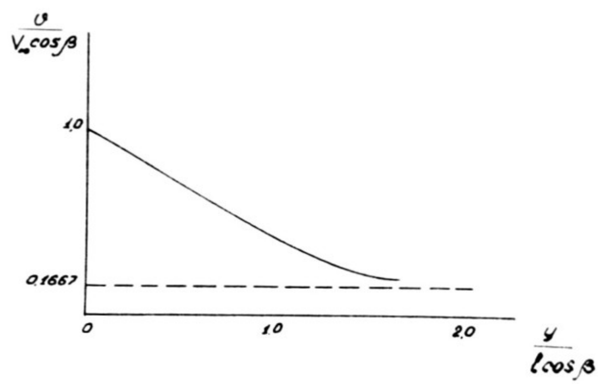

Fig. 2.

independent of ρ_∞ and C, it is obvious that we have to deal with a self-similar problem. To eliminate the symbol for the mass unit, we introduce the ratio ρ_∞/C which has the dimensionality

$$\left[\frac{\rho_\infty}{C}\right] = \frac{L^{2n-2}}{T^{2n-1}} \tag{10}$$

Self-similar, would then be the following cases:

(a) the uniformly accelerated motion of a thermally insulated, or absolutely cold, body without a characteristic length (plate, cone, wedge) if $n = 3/2$; since in this case the ratio [Eq. (10)] takes on the dimensionality of the acceleration $[a] = LT^{-2}$;

(b) the rotation of thermally insulated, or absolutely cold, conical axially symmetric surfaces if $n = 1$; since, in this case, the ratio [Eq. (10)] assumes the dimensionality of the angular velocity $[\Omega] = T^{-1}$.

(c) a point explosion with central symmetry if $n = 1/6$; since, in this case, Eq. (10) has the dimensionality

$$\left[\frac{\rho_\infty}{E}\right]^{1/3} = \frac{L^{-5/3}}{T^{-2/3}}$$

E being the energy of the explosion.

Let us examine in detail just one problem of the first type—namely that of a gas which flows under the influence of a uniformly accelerated infinite plate.

UNIFORMLY ACCELERATED MOTION OF AN INFINITE PLATE

The nonsteady motion of a gas under the influence of an infinite plate parallel to the x axis, depends in the general case upon the two independent variables y and t. The system of equations which describes such a motion has the form:

$$\begin{gathered}
\rho\left(\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y}\right) = \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) \\
\rho\left(\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y} + \frac{4}{3}\frac{\partial}{\partial y}\left(\mu\frac{\partial v}{\partial y}\right) \\
\rho\left(\frac{\partial h}{\partial t} + v\frac{\partial h}{\partial y}\right) = \frac{\partial p}{\partial t} + v\frac{\partial p}{\partial y} + \frac{1}{\sigma}\frac{\partial}{\partial y}\left(\mu\frac{\partial h}{\partial y}\right) \\
+ \mu\left(\frac{\partial u}{\partial y}\right)^2 + \frac{4}{3}\mu\left(\frac{\partial v}{\partial y}\right)^2 \\
\frac{\partial \rho}{\partial t} + \frac{\partial \rho v}{\partial y} = 0, \qquad p = \frac{\gamma - 1}{\gamma}\rho h
\end{gathered} \tag{11}$$

On the basis of the discontinuity equation, we introduce the function χ, defined by the relations

$$\frac{\partial \chi}{\partial t} = -\rho v, \qquad \frac{\partial \chi}{\partial y} = \rho \tag{12}$$

and therefore the system of equations (11) from the independent variables y, t to the independent variables χ, t.

$$
\begin{aligned}
\frac{\partial u}{\partial t} &= \frac{\partial}{\partial \chi}\left(\mu\rho \frac{\partial u}{\partial \chi}\right) \\
\frac{\partial v}{\partial t} &= -\frac{\partial p}{\partial \chi} + \frac{4}{3}\frac{\partial}{\partial \chi}\left(\mu\rho \frac{\partial v}{\partial \chi}\right) \\
\frac{\partial h}{\partial t} &= \frac{1}{\rho}\frac{\partial p}{\partial t} + \frac{1}{\rho}\frac{\partial}{\partial \chi}\left(\mu\rho \frac{\partial h}{\partial \chi}\right) + \frac{4}{3}\mu\rho\left(\frac{\partial v}{\partial \chi}\right)^2 + \mu\rho\left(\frac{\partial u}{\partial \chi}\right)^2 \\
\frac{\partial y}{\partial \chi} &= \frac{1}{\rho}, \qquad \frac{\partial y}{\partial t} = v, \qquad p = \frac{\gamma - 1}{\gamma}\rho h
\end{aligned}
\tag{13}
$$

Let the plate move at a constant acceleration vector $\vec{a}$ which is directed to its plane at an arbitrary angle of incidence α (Fig. 3). In conformity with what has been said above, we assume the dependence of the viscosity coefficient upon enthalpy to have the form

$$\mu = Ch^{3/2} \tag{14}$$

The determinant parameters of the problem will then lack the constants having the dimensionalities of length and time. The only possible nondimensional combination between the independent variables χ, t and these parameters, will be

$$z = \frac{\chi}{\rho_\infty a t^2} \tag{15}$$

so that the solution of the problem under discussion may be given in the form:

$$
\begin{aligned}
u &= atU(z) & \rho &= \rho_\infty R(z) \\
v &= atV(z) & h &= a^2t^2H(z) \\
p &= \rho_\infty a^2t^2P(z) & y &= at^2Y(z)
\end{aligned}
\tag{16}
$$

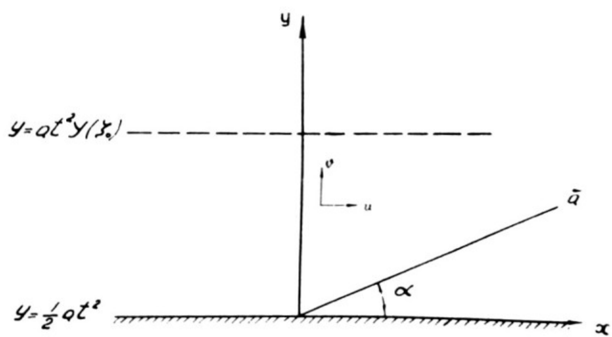

Fig. 3.

Substituting these expressions into the system of equations (13), we obtain for the dimensionless variables the following system of ordinary differential equations.

$$
\begin{aligned}
&U - 2zU' = k(RH^{3/2}U')' \\
&V - 2zV' = -P' + \frac{4}{3}k(RH^{3/2}V')' \\
&2R(H - zH') = 2(P - zP') + \frac{k}{\sigma}R(RH^{3/2}H')' \\
&\qquad\qquad + kR^2H^{3/2}\left(U'^2 + \frac{4}{3}V'^2\right)
\end{aligned}
\tag{17}
$$

$$R^2V' = 2zR', \qquad P = \frac{\gamma - 1}{\gamma}RH, \qquad Y = \frac{V}{2} + \frac{z}{R} \tag{18}$$

where the dimensionless parameter

$$k = \frac{C_a}{\rho_\infty} \tag{19}$$

is inversely proportional to the Reynolds number of the problem; the primes denote differentiation with respect to z.

Let us examine the system of boundary conditions. At the plate surface we have

$$z = 0, \qquad U = \cos\alpha, \qquad V = \sin\alpha \tag{20}$$

as well as one of the two conditions:

$$H' = 0 \tag{21}$$

for an insulated plate, or

$$H = 0 \tag{22}$$

for the case of cold surface.

Assume now that the region of perturbed motion is separated from the remaining part of the space by a front surface that propagates at a constant acceleration. At the surface of such a front we then have:

$$z = z_0, \qquad U = V = P = H = 0, \qquad R = 1 \tag{23}$$

where the constant z_0 is to be determined. The rest of the problem consists in the integration of the system [Eq. (17)] with the boundary conditions [Eqs. (20) through (23)].

To solve this problem, it is first necessary to examine the behavior of the solution of system [Eq. (17)] near the front at $z \to z_0$, since in this case this is a singular solution. As has been said above, this can be accomplished by postulating constant the propagation velocity of the front during a sufficiently short period of time. Then we may use the approximations derived in the first section of this paper for the components of velocity, enthalpy, pressure, and density [Eq. (8)].

The transition from the variables in these formulas to the variables [Eqs. (15), (16)] is readily performed by stating that in conformity with Eqs. (15), (18), and (23), the propagation velocity of the front is equal to

$$c = \frac{d}{dt}\left[at^2 y(z_0)\right] = 2atz_0 \tag{24}$$

After some simple transformations, the expressions [Eq. (8)] (at $n = 3/2$) then yield the following approximations for the unknown functions at $z \to z_0$:

$$\begin{aligned}
U &\cong A(z_0 - z)^{2\gamma/3\sigma} \\
V &\cong \frac{1}{2z_0}\left(\frac{3\sigma}{k\gamma} z_0\right)^{2/3} \frac{\gamma - 1}{\gamma - \frac{4}{3}\sigma} (z_0 - z)^{2/3} + B(z_0 - z)^{(1/2)(\gamma/\sigma)} \\
H &\cong \left(\frac{3\sigma}{k\gamma} z_0\right)^{2/3} (z_0 - z)^{2/3} \\
R &\cong 1 + \frac{1}{2z_0} V, \qquad P \cong \frac{\gamma - 1}{\gamma} H
\end{aligned} \tag{25}$$

The constants z_0, A, and B in these formulas are to be determined. Their number, of course, is equal to that of the boundary conditions [Eqs. (20) through (22)] at $z = 0$.

Thus, the solution of the problem under examination, at a small distance from the front, is given by the approximations [Eq. (25)], and from this point on can be set forth by a numerical method. However, in practice, such a method of calculation becomes extremely cumbersome due to the necessity of determining the three arbitrary constants z_0, A, and B on the basis of the boundary conditions at the opposite end of the interval. These difficulties can be appreciably reduced by making use of the invariability of the system [Eq. (17)] with regard to the transformation form:

$$\begin{array}{ll|ll}
U &= \lambda\hat{U} & R &= \hat{R} \\
V &= \lambda\hat{V} & z &= \lambda\hat{z} \\
H &= \lambda^2\hat{H} & k &= \dfrac{\hat{k}}{\lambda}
\end{array} \tag{26}$$

where λ is an arbitrary constant. By selecting this constant in the form

$$\lambda = \left(\frac{3\sigma}{k\gamma}\right)^{1/3} {z_0}^{2/3} \tag{27}$$

the expressions [Eq. (25)] are reduced to the form

$$\begin{aligned}
\hat{U} &\cong \hat{A}\left(1 - \frac{\hat{z}}{z_0}\right)^{2\gamma/3\sigma} \\
\hat{V} &\cong \frac{1}{2z_0}\,\frac{\gamma - 1}{\gamma - \frac{4}{3}\sigma}\left(1 - \frac{\hat{z}}{z_0}\right)^{2/3} + \hat{B}\left(1 - \frac{\hat{z}}{z_0}\right)^{(1/2)(\gamma/\sigma)} \\
\hat{H} &\cong \left(1 - \frac{\hat{z}}{z_0}\right)^{2/3} \\
\hat{R} &\cong 1 + \frac{1}{2z_0}\hat{V}, \qquad \hat{\rho} \cong \frac{\gamma - 1}{\gamma}\hat{H}
\end{aligned} \tag{28}$$

here, Eq. (27) yields

$$\hat{z}_0 = \sqrt{\frac{\gamma}{3\sigma}\hat{k}} \tag{29}$$

In this way, two of the three independent parameters $\hat{A}$, $\hat{B}$, and $\hat{k}$ (say $\hat{A}$ and $\hat{K}$) can be now taken arbitrarily, and only one ($\hat{B}$) must be determined, either from the condition [Eq. (21)] or [Eq. (22)]. Such a simplification of the problem, however, does not allow to prescribe a priori the Reynolds number and the angle of incidence on the plate. These are found as a result of integration. To determine the angle of incidence we make use of the relation

$$\alpha = \arctan\frac{\hat{V}(0)}{\hat{U}(0)} \tag{30}$$

which follows from the boundary conditions [Eq. (20)]. The same boundary conditions serve as a basis for the determination of the transformation parameter [Eq. (27)]:

$$\lambda = \frac{1}{\sqrt{\hat{U}^2(0) + \hat{V}^2(0)}} \tag{31}$$

whereafter all initial functions as well as the values of the parameters z_0 and k can be determined.

NUMERICAL RESULTS

The described method for the solution of the problem was used in the numerical calculations of the flows near a thermally insulated and a cold plate. The system of differential equations was integrated by the Runge-Kutta method, using an electronic computer. The program was developed for automatic selection of the step with respect to a given accuracy, the latter being chosen as

0.01 percent. The calculations were performed for two values of the Reynolds number that were in correspondence with the parameter values $\hat{k} = 1$ and $\hat{k} = 0.1$. The values of the adiabatic exponent and the Prandtl number were taken as $\gamma = 1.4$ and $\sigma = 0.70$.

It should be noted that a further decrease in the value of the parameter k (increase in Reynolds number) greatly complicates the calculations due to the market variation of all functions with decreasing thickness of the shock-wave layer.

Some results of calculations carried out for near-zero angles of incidence are given in Figs. 4 through 10 in the form of the dependence of the unknown functions upon the variable

$$\eta = \frac{Y - Y(0)}{Y(z_0) - Y(0)} \tag{32}$$

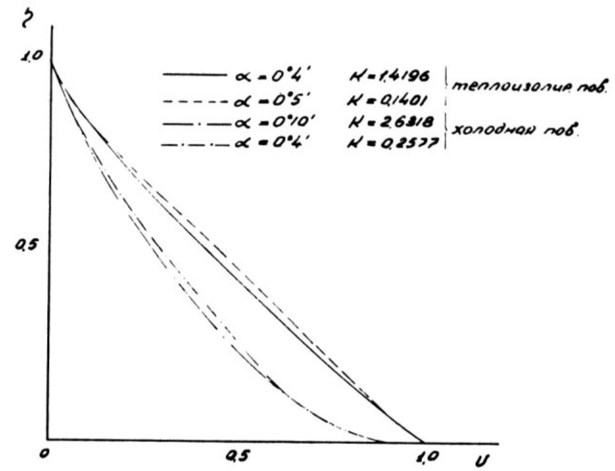

Fig. 4.

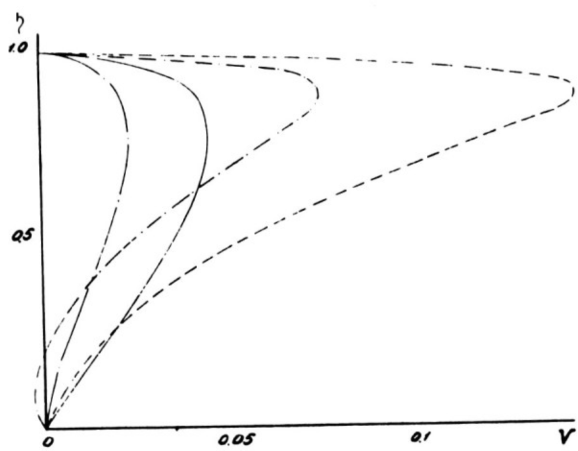

Fig. 5.

From the analysis of the obtained relations it is seen that a change in Re has practically no effect upon the temperature profile in the perturbed region (Fig. 6), and only slight affects on the changes of the longitudinal velocity component (Fig. 4). The cooling of the plate affects to a great extent the character of the behavior of all functions of the flow field. It is of interest to note, however, that on the density profile, this effect is concentrated in a very thin layer at surface, where the density of the gas undergoes an unlimited increase while the temperature tends to zero (Figs. 6, 7). The surface of the absolutely cold plate is the singular point of the solution. To illustrate this singularity‡ we employ first of all the condition of the finiteness of the heat flow through the unit of plate-surface area, which yields

$$H^{3/2}\frac{dH}{d\eta} \sim \eta^{0} \qquad \text{or} \qquad H \sim \eta^{2/5} \tag{33}$$

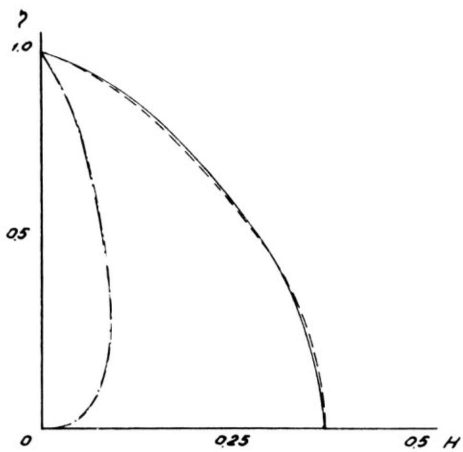

Fig. 6.

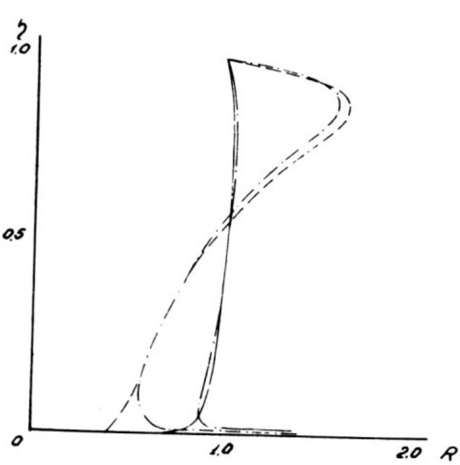

Fig. 7.

From the condition of the finiteness of friction stress on this surface, we have:

$$H^{3/2}\frac{dU}{d\eta} \sim \eta^{0} \qquad \text{or} \qquad 1 - U \sim \eta^{2/5} \tag{34}$$

Since the pressure at the surface $\eta = 0$ is also finite, the equation of state yeilds the law of the change in density at $\eta \to 0$:

$$R \sim \eta^{-2/5} \tag{35}$$

Further, from the continuity equation it may be concluded that the vertical velocity component varies proportionally to η:

$$V \cong V'(0)\eta \tag{36}$$

To determine the character of the change in pressure, it is necessary to use the equation which after transformation to the independent variable η can be written in the form:

$$R(V - 2\hat{\eta}V' + VV') = -P' + \frac{4}{3}k\,(H^{3/2}V')' \tag{37}$$

where $\hat{\eta} = z_0\eta$, and the primes denote differentiation with respect to this variable. Using the results obtained, we find that the change in pressure in a narrow layer at the plate surface is defined by the "viscous" term [Eq. (37)], so that at $\eta \to 0$:

$$P - P(0) \sim V'(0)\eta^{2/5} \tag{38}$$

Hence, the pressure near the greatly cooled surface of a plate that moves in its own plane can withstand extreme changes. This forbids the use of the boundary-layer equation for the investigation of the flow in this region. With increasing distance from the plate surface, the relative effect of the viscosity decreases rapidly, while the influence of the inertia terms [Eq. (37)] increases. Already at quite small distances η, the change in pressure will be defined by the inertia terms. As a result, we obtain a behavior of the function $P(\eta)$ as shown in Fig. 8, and also, in more detail, for small values of η in Fig. 9.

The initial decrease in P is associated with the existence of a region near the plate surface, where the gas moves toward the surface, i.e., the constant $V'(0)$ included in Eq. (30) is negative (Figs. 5, 10).

It should be noted that the described singular character of the behavior of the solution near the surface of the cold plate is not a consequence of the small magnitude of the Reynolds numbers assumed in the calculations, since in the derivation of Eqs. (33) through (38) this assumption did not take place. But these results refer to a region of the flow which is so narrow that the applicability of the equations of continuum mechanics to this flow is possibly not fully justified.

The analysis of the results obtained for the region adjacent to the front ($\eta = 1$) shows that with increasing Reynolds number, the normal velocity component, the pressure, and density assume here increasingly larger external values (see Figs. 5, 7, 8). Speaking generally, a property common to all the flows examined is that for the very small values of the Reynolds number assumed in the calculations, the shock wave does not form, so that neither the pressure nor the temperature attain anywhere the values that correspond to a full drop across the shock wave.

In conclusion, we might note that the exact solution which has been obtained for the problem of the uniformly accelerated motion of an infinite plate is of interest for the evaluation of various approximate methods of analysis. Thus,

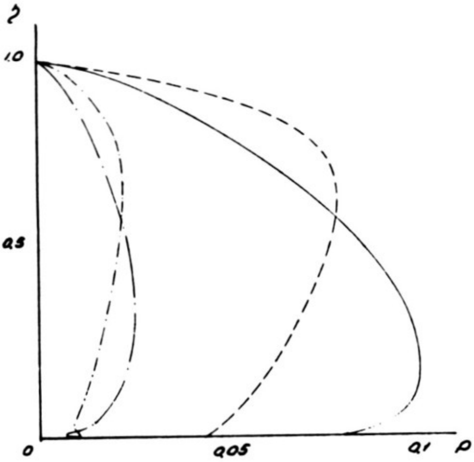

Fig. 8.

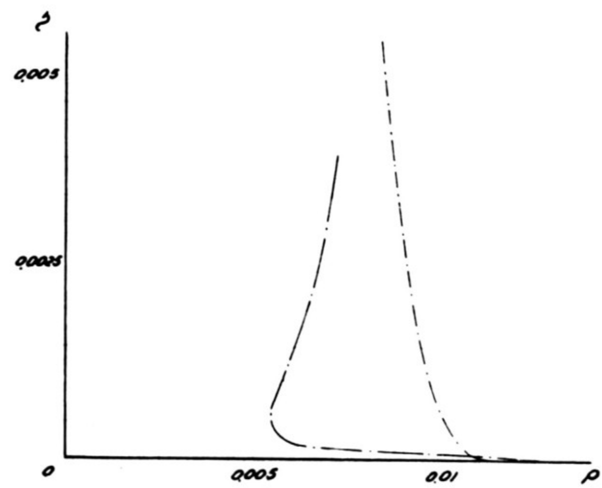

Fig. 9.

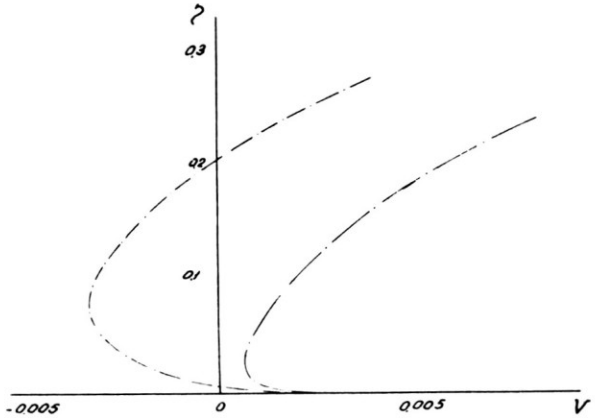

Fig. 10.

for example, it would be of interest to assess the accuracy of the theory of strong interaction between the boundary layer and the external flow, and also the accuracy of the approximate calculations of the structure of nonsteady shock waves.

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